The Vague, the Assertable, and the Omega-Knowable*

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Abstract

It is widely accepted that knowledge is necessary for proper assertion. More controversial is the thesis that *omega*-knowledge—infinitely higher-order knowledge—is necessary for proper assertion. Proponents of the "KK" principle take the former thesis to entail the latter, since they take knowledge to entail omega-knowledge. But in *Iterated Knowledge*, Simon Goldstein both argues against the KK principle and in favor of the thesis that omega-knowledge is necessary for proper assertion. This paper argues that reflection on some independently plausible principles about the relationship between knowledge, assertion, and vagueness shows that Goldstein and the proponents of KK are both mistaken. First: if (as is widely accepted) vagueness is a barrier to knowledge, then one can't omega-know that it's vague what one omega-knows. But if omega-knowledge is plentiful, then it's clearly assertable (and thus knowable) that sometimes it's vague what one omega-knows. So if vagueness is a barrier to knowledge, then omega-knowledge can't be necessary for proper assertion, and knowledge can't entail omega-knowledge. Second: even if vagueness isn't a barrier to knowledge, it's a barrier to proper assertion. But this presents a dilemma for Goldstein: either being in a position to assert something entails being in a position to assert that one is in a position to assert it, or it doesn't. If it does, then one can't assert that it's sometimes vague what one is in a position to assert. But that looks like a problem, because it seems clear that one can assert that sometimes it's vague what one is in a position to assert. But if being in a position to assert something doesn't entail being in a position to assert that one is in a position to assert it, then the principle that assertion requires omega-knowledge fails to do the explanatory work Goldstein needs it to do. Either way, we should reject arguments for omega-knowledge anti-skepticism grounded in judgments about proper assertion.

1 Introduction

Omega-knowledge is infinitely higher-order knowledge. If one omega-knows p, then one knows p, knows that one knows p, knows that one knows that one knows p, and so on *ad infinitum*. What, if anything, do we omega-know?

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According to the omega-knowledge skeptic, the answer is: very little. Perhaps certain special domains lend themselves to omega-knowledge: logic, mathematics, conceptual matters, the cogito, and so on. But that's *trivial* omega-knowledge. Non-trivial omega-knowledge is where the action is—that is: omega-knowledge of ordinary, contingent propositions, like those we come to know on the basis of perception and testimony. And according to the omega-knowledge skeptic, non-trivial omega-knowledge is impossible. Even if I omega-know that $2+2=4$ or that all bachelors are unmarried, I don't omega-know what year it is, whether I'm looking at a screen, or how many hands I have.[1]

The omega-knowledge anti-skeptic says otherwise: non-trivial omega-knowledge is plentiful. Some of these anti-skeptics accept the "KK" principle, and so believe that non-trivial omega-knowledge is exactly as plentiful as non-trivial first-order knowledge (which, let's assume, is plentiful).

But there is at least one omega-knowledge anti-skeptic who rejects the "KK" principle: Simon Goldstein. According to Goldstein (2024, Ch.1), proper assertion requires omega-knowledge, so we omega-know whatever we can properly assert. Since I can properly assert that it's 2025, that I'm looking at a screen, and that I have exactly two hands, I must omega-know each of these things. So non-trivial omega-knowledge must be possible. More generally, since proper assertion is plentiful, omega-knowledge is too.

Why does Goldstein think that omega-knowledge is necessary for proper assertion? Here's the rough idea. First-order knowledge seems necessary for proper assertion, as the existence of such a norm neatly explains the infelicity of "Moorean conjunctions" like 'p but I don't know that p'. You can't know that p and that you don't know that p, so, given the first-order knowledge norm, you can't assert it either. But such a norm by itself is powerless to explain the infelicity of "dubious conjunctions" like 'p but I don't know whether I know that p', since, for all that's been said, you *can* know both that p and that you don't know whether you know that p. But, crucially, you can't know that you know this. So if there were a second-order knowledge norm on assertion ('Assert that p only if you know that you know p'), then we'd be able to account for these dubious conjunctions too. However, we'd still lack an explanation of dubious conjunctions at higher levels of iteration, like 'p but for all I know I don't know whether I know that p', since such claims are knowably knowable. But, crucially, they're not knowably knowably knowable! So perhaps what we really need is a *third*-order knowledge norm on assertion. But then what about 'p, but for all I know I don't know whether I don't know whether I know that p'...?

We have a vertiginous hierarchy of iterated dubious conjunctions. According to Goldstein (2024, pp.10-11), they are infelicitous at every order, and at no point is their infelicity due merely to difficulties in comprehension. If the norm on proper assertion is omega-knowledge—knowledge at every order—then we cleanly predict that any such construction is infelicitous. Anything less and it's not clear we have a good story to tell. So, Goldstein concludes, proper assertion requires omega-knowledge.[2]

[1] I won't rehearse the standard arguments for omega-knowledge skepticism. Interested readers can consult Goldstein's (2024, Ch.1) helpful survey.

[2] This style of argument has also been advanced by Sosa (2009), Cohen & Comesaña (2013), Greco (2014), Das & Salow (2018), Dorst (2019), and Haziza (2022). For some existing criticisms, see Benton (2013), Williamson (2013), and Mandelkern & Dorst (2022).

That in turn sets up the argument for omega-knowledge anti-skepticism. Given that omega-knowledge is the norm on assertion, if we're omega-knowledge skeptics, then we must also be assertion skeptics—people who think little of what we ever assert is properly assertable. But, *pace* Unger (1975), assertion skepticism is untenable. So we should reject omega-knowledge skepticism.[3]

I am an omega-knowledge skeptic. But I am not an assertion skeptic: I think much of what we assert we assert properly. So I am committed to thinking that Goldstein's assertion-based argument for omega-knowledge anti-skepticism goes wrong somewhere.

I'm not *exactly* certain where the argument goes wrong—and, unfortunately, considerations of space prevent me from engaging with this question in the detail it deserves. Still, I am all but certain that Goldstein's argument is unsound. The issue, in a nutshell, is that the claim that proper assertion requires omega-knowledge plays rather badly with some independently plausible principles about the relationship between knowledge, assertion, and vagueness. I'll present two arguments that bring out the tension: one that assumes that vagueness entails ignorance, and one that doesn't.

The first argument will establish that we can't omega-know that it's sometimes vague what we omega-know. This is a problem for Goldstein, because if omega-knowledge is possible (let alone quotidian, as he claims), then it's clearly assertable that sometimes it's vague what we omega-know. But if it's clearly assertable that it's sometimes vague what we omega-know, and yet we can't omega-know that it's sometimes vague what we omega-know, then, *pace* Goldstein, it can't be that omega-knowledge is necessary for assertion.

The second argument takes the form of a dilemma: either being in a position to assert something entails being in a position to assert that one is in a position to assert it, or it doesn't. If it does, then we can't assert that it's sometimes vague what we're in a position to assert. But that looks like a problem, because it seems clear we *can* assert that sometimes it's vague what we're in a position to assert. But if being in a position to assert something *doesn't* entail being in a position to assert that one is in a position to assert it, then the principle that assertion requires omega-knowledge does essentially nothing to prevent the proliferation of dubious assertions.

So, I conclude: the principle that assertion requires omega-knowledge is either false or an idle wheel. Does this mean we should be omega-knowledge skeptics? Not necessarily. But it does suggest that if we're going to be omega-knowledge anti-skeptics, it shouldn't be on the basis of judgments about proper assertion.[4]

[3] In fact, Goldstein (Ch.1) argues that we omega-know any p such that there is some act or state Φ such that: (i) we properly Φ and (ii) there is a norm that says Φ-ing is proper only if p. (Assertion is just a special case of this general rule, where the relevant norm is that the content of one's assertion be true.) The argument relies crucially on a "Norm Iteration" principle, which says that if you are permitted to Φ only if p, then you are permitted to Φ only if you know that p. For the sake of concreteness I'll only be focusing on Goldstein's assertion-based argument for anti-skepticism about omega-knowledge, but as far as I can tell my objections apply just as well to the more general argument, suggesting that Norm Iteration cannot be true.

[4] We'll also see that considerations of vagueness set up a direct argument against the "KK" principle, thereby cutting off that route to anti-skepticism as well.

2 Background assumptions

Start with a paradigm case of vagueness. A glass starts out 100% full and is continuously emptied to the point of being 0% full. At the start of the process, the glass is pretty full; by the end, it's not.[5] When exactly does the glass stop being pretty full?

I have no idea how one is supposed to answer this question. It seems the best one can do is say that it's *vague* when the glass stops being pretty full.[6] In fact, its being vague when the glass stops being pretty full seems to be a perfectly good explanation of *why* we're in no position to say exactly when the glass stops being pretty full.

Though I will be making number of assumptions about the logic of vagueness (more on which in a moment), I will not be presupposing any particular theory of vagueness in this paper. I will be taking the notion as a primitive, understood by ostension to cases like the one just described. And I will be understanding the notion of *determinacy* derivatively: for it to be determinate that p is for it to be the case that: p and it's not vague whether p.

With this in mind, here is some of the notation I'll be using in what's to come:

Vagueness

- $\nabla p :=$ it is vague whether p.
- $\Delta p := p \wedge \neg \nabla p$. ('It is determinate that p')
- $\Delta^n p := \underbrace{\Delta \ldots \Delta}_{n} p$. ('It is n-determinate that p'.)
- $\Delta^* p := p \wedge \Delta p \wedge \Delta\Delta p \wedge \ldots := \underbrace{\Delta \ldots \Delta}_{\infty} p$. ('It is omega-determinate that p'.)

Knowledge

- $Kp :=$ it is knowable that p.[7]
- $\Diamond p := \neg K \neg p$. ('For all that is knowable, p'.)
- $K^n p := \underbrace{K \ldots K}_{n} p$. ('It is n-knowable that p'.)
- $K^* p := p \wedge Kp \wedge KKp \wedge \ldots := \underbrace{K \ldots K}_{\infty} p$. ('It is omega-knowable that p'.)

And here are some of the general assumptions about vagueness and knowledge I'll be employing throughout the paper:

- As implied by the notation just introduced, I will be assuming a propositional conception of vagueness—i.e., that vagueness and determinacy are properties of propositions. Propositional vagueness is fairly controversial, so I'll simply refer readers

[5] Granted, saying that a glass that is 100% full is "pretty full" is a bit of an understatement. Those who find this distracting are invited to read 'pretty full' as 'at least pretty full'.

[6] It's probably also context-sensitive, with the interpretation of 'pretty full' being sensitive to the purposes of the glass and its contents (*inter alia*), but I'll suppress that in what follows.

[7] In general my discussion will be centered on the somewhat abstract notion of *knowability*, which I intend to be understood as a kind of "idealized" knowledge. This is to avoid annoying issues that arise from failures of logical omniscience and the like. As a rough heuristic, the claim that p is knowable can be understood as the claim that p could be known by a logically omniscient agent. But note that this gloss must be taken with a grain of salt, since (*inter alia*) there may well be actually false propositions that are knowable in this sense (e.g., the proposition that there is a logically omniscient agent). See Yli-Vakkuri & Hawthorne (2022) for a survey of some of the issues here.

to Bacon (2018) for a book-length defense of its coherence (and significance). As far as I can tell, the paper's central arguments could be run on the assumption that vagueness and determinacy are properties of sentences, though the complexity of the discussion would get out of hand very quickly.

- I will be assuming classical logic. This assumption is also fairly controversial in the present setting. But (a) it's plausible enough in its own right and (b) it would be surprising if Goldstein's assertion-theoretic argument for anti-skepticism about omega-knowledge was implicitly committed to a rejection of classical logic. So I won't be saying more in its defense.

- I will be assuming that both Δ and K obey (at least) the normal modal logic KT, and thus that:[8]

  Δ-CLOSURE: $\Delta(p \rightarrow q) \rightarrow (\Delta p \rightarrow \Delta q)$
  Δ-FACTIVITY: $\Delta p \rightarrow p$

  K-CLOSURE: $K(p \rightarrow q) \rightarrow (Kp \rightarrow Kq)$
  K-FACTIVITY: $Kp \rightarrow p$

- I will be assuming that the classical conjunction introduction and elimination rules extend to the infinitary case, so that from $p_1, p_2, \ldots$ one may infer $p_1 \wedge p_2 \ldots$, and from $p_1 \wedge p_2 \wedge \ldots$ one may infer $p_1, p_2, \ldots$, and so on.

- I will be assuming that determinacy and knowledge both agglomerate over (infinite) conjunctions:

  Δ-AGGLOMERATION: $(\Delta p_1 \wedge \Delta p_2 \wedge \ldots) \rightarrow \Delta(p_1 \wedge p_2 \wedge \ldots)$
  K-AGGLOMERATION: $(Kp_1 \wedge Kp_2 \wedge \ldots) \rightarrow K(p_1 \wedge p_2 \wedge \ldots)$

  Which, given our other assumptions, entails that both omega-determinacy and omega-knowledge obey positive iteration principles:[9]

  Δ^*-POSITIVE: $\Delta^* p \rightarrow \Delta\Delta^* p$
  K^*-POSITIVE: $K^* p \rightarrow KK^* p$

- I will be assuming that anything that is a logical truth is thereby both omega-determinate and omega-knowable (where $\blacksquare p :=$ it is a logical truth that p):[10]

  Δ^*-NECESSITATION: $\blacksquare p \rightarrow \Delta^* p$
  K^*-NECESSITATION: $\blacksquare p \rightarrow K^* p$

[8] Note that Δ-FACTIVITY follows from the definition of 'Δ'.

[9] *Proof* of Δ^*-POSITIVE (taken from Bacon 2020, who cites Williamson 1994): Suppose $\Delta^* p$. Then we have $p \wedge \Delta p \wedge \Delta\Delta p \wedge \ldots$, which in turn entails $\Delta p \wedge \Delta\Delta p \wedge \ldots$. Given Δ-AGGLOMERATION, this entails $\Delta(p \wedge \Delta p \wedge \Delta\Delta p \wedge \ldots)$. And that's just $\Delta\Delta^* p$. The proof of K^*-POSITIVE is exactly analogous. (Goldstein himself accepts K^*-POSITIVE on essentially these grounds.)

[10] There are some, like Dorr (2010), who deny Δ^*-NECESSITATION. But for the purposes of this paper we needn't concern ourselves with such a perspective, since one who denies Δ^*-NECESSITATION has independent reasons to reject Goldstein's views on omega-knowledge and assertion. To give a sense of the issue: if Δ^*-NECESSITATION fails, then there is reason to think K^*-NECESSITATION fails (see note 14 below). But if K^*-NECESSITATION fails, then this is bad news for Goldstein, since logical truths are assertable, if nothing else.

- I will be assuming that each of the assumptions listed so far is itself a logical truth, at least in the sense of 'logical truth' relevant to the intended interpretation of the NECESSITATION principles. Thus, each of these assumptions is both omega-determinate and omega-knowable.

Finally, since the main goal of this paper is to refute Goldstein's principle connecting assertion to omega-knowledge (where $Ap :=$ it is assertable that p)—

OMEGA ASSERTION: $Ap \to K^*p$

—I will be assuming it (for the purposes of a reductio) in what follows.[11] With that we may now proceed to the arguments.

3 Vagueness with ignorance

A widely held view in the philosophy of vagueness is that vagueness—or, really, a lack of determinacy—entails ignorance:

IGNORANCE: $\neg\Delta p \to \neg Kp$

I will not be defending IGNORANCE at length here. It's a dominant view in the literature, and the arguments for and against it have been discussed in detail elsewhere.[12] But the intuitive motivation for it is simple. Consider the emptying glass. If I ask you whether you know when the glass stops being pretty full, 'No' is a *much* more natural answer than 'Yes'. Relatedly, if you did know at what point the glass stops being pretty full, then what's stopping you from saying what it is?[13] It's just quite hard to see how, despite the appearances, one could know at what exact percentage of fullness a glass stops being pretty full.

So I'm going to assume for now that IGNORANCE is true. I'm also going to assume the following conditional: *if* IGNORANCE is true, then it's assertable. Though there are some who deny IGNORANCE, I don't know anyone who has considered denying *this* claim. Given OMEGA ASSERTION, the assumption that IGNORANCE is assertable entails that IGNORANCE is omega-knowable.[14]

[11] Note that I will not be assuming that OMEGA ASSERTION is a *logical* truth. The assumption of its truth is enough for the reductio.

[12] For arguments in favor of IGNORANCE, see, e.g., Sorensen (1988), Williamson (1994), Horwich (1997), Lee (2016), and Bacon (2018, Ch.5). For arguments against IGNORANCE, see, e.g., Wright (2001), Dorr (2003), and Barnett (2010).

[13] Dorr (2003) argues that we can give a good answer to this question without invoking IGNORANCE—see note 28 below.

[14] Note that a consequence of the omega-knowability of IGNORANCE is that a failure of omega-determinacy entails a failure of omega-knowabillity:

OMEGA IGNORANCE: $\neg\Delta^*p \to \neg K^*p$

Proof: Suppose $\neg\Delta^*p$. Then, for some n, $\neg\Delta^n p$. So $\neg\Delta\Delta^{n-1}p$. By IGNORANCE, this entails $\neg K\Delta^{n-1}p$. So $\Diamond\neg\Delta^{n-1}p$. So $\Diamond\neg\Delta\Delta^{n-2}p$. But given that IGNORANCE is omega-known (and thus 2-known), this entails $\Diamond\neg K\Delta^{n-2}p$. And that's just: $\Diamond^2\neg\Delta^{n-2}p$. Continuing on, we eventually get to $\Diamond^n\neg p$, which is just $\neg K^n p$. And that entails $\neg K^*p$.

Apropos of note 10 above, it's worth emphasizing that OMEGA IGNORANCE implies that if Δ^*-NECESSITATION fails, then so does K^*-NECESSITATION. But if K^*-NECESSITATION fails, then some logical truths are not omega-knowable. One who accepts both IGNORANCE and OMEGA ASSERTION thus has strong reasons to accept Δ^*-NECESSITATION, since being a logical truth sure seems like it ought to suffice for being assertable.

Now we have the materials to start raising some challenges to Goldstein's views. I'll start with one that I think comes pretty close to begging the question, but that will serve as an instructive warmup to the more serious problems.

Return to the case of the emptying glass, and consider the following principle: if it's determinately true that a glass that is x% full is pretty full, then it's true that a glass that is $(x-1)$% full is pretty full. That is (where $p_x :=$ the proposition that a glass that is x% full is pretty full):[15]

Δ-MARGIN: $\Delta p_x \rightarrow p_{x-1}$

Though admittedly a bit abstract, this principle has quite a lot of initial appeal: how could it be *determinately* true—and thus not vague—that a glass that is (say) 80% full is pretty full, while a glass that is 79% full—only barely less full!—isn't even pretty full? It's natural to expect a certain amount of gradualness in the cup's transition from being determinately pretty full to being not pretty full, with it spending at least *some* amount of time being non-determinately pretty full before becoming (presumably non-determinately) not pretty full.[16]

So let's suppose Δ-MARGIN is true. In fact, let's suppose it's assertable, as it seems to be (at least assuming it's actually true). Then, by OMEGA ASSERTION, it's omega-knowable.

But notice that, given IGNORANCE, Δ-MARGIN entails a "margin for error" principle for knowledge (cf. Williamson 2000):[17]

K-MARGIN: $Kp_x \rightarrow p_{x-1}$

Which itself must be both omega-knowable and omega-determinate, at least assuming that both IGNORANCE and Δ-MARGIN are.

Finally, note that it's clearly assertable that a glass that is 100% full is pretty full (p_{100}) and that a glass that is 0% full is not pretty full ($\neg p_0$). And so, given OMEGA ASSERTION, both of these claims are omega-knowable too.

But now we've fallen into inconsistency (cf. Gómez-Torrente 1997, Zardini 2013b):[18]

1. $K^*(Kp_x \rightarrow p_{x-1})$ (IGNORANCE, Δ-MARGIN)
2. K^*p_{100} (Assumption)
3. $K^*(Kp_{100} \rightarrow p_{99})$ (1)
4. $K^*Kp_{100} \rightarrow K^*p_{99}$ (3)
5. $K^*p_{100} \rightarrow K^*p_{99}$ (4)

[15] For discussion of principles like Δ-MARGIN, see, e.g., Williamson (1994), Gómez-Torrente (2002), and Zardini (2013b).

[16] In case it helps, one is invited to interpret the '1' in Δ-MARGIN as (say) a basis point rather than a full percent. Indeed, the relevant numbers can be made arbitrarily small, so long as they remain fixed. (Similar points apply to the other principles I will be introducing throughout the paper.)

[17] Proof: Suppose Kp_x. By IGNORANCE, Δp_x. So by Δ-MARGIN, p_{x-1}.

[18] Here and below I sometimes justify a step of a proof by citing a principle (say IGNORANCE, or Δ-MARGIN), when in fact the relevant assumption is that the principle in question is omega-knowable (see, e.g., step 1 of this first proof). Since my interest here is in showing that certain principles are not co-omega-knowable, I think this slight sleight of hand is justified by considerations of presentational austerity.

6. $K^* p_{99}$ (2, 5)

7. $K^*(K p_{99} \rightarrow p_{98})$ (1)

...

402. $K^* p_0$ (398, 401)

403. $K^* \neg p_0$ (Assumption)

404. $\neg K^* p_0$ (403)

405. $\perp$ (402, 404)

Thus, as a matter of (classical) logic, at least one of the following four claims is not omega-knowable:

- A glass that is 100% full is pretty full. (p_{100})
- A glass that is 0% full is not pretty full. ($\neg p_0$)
- Vagueness entails ignorance. (IGNORANCE)
- If it's determinate that a glass that is x% full is pretty full, then a glass that is $(x-1)$% full is pretty full. (Δ-MARGIN)

Since p_{100} and $\neg p_0$ look like logical truths, I'm going to set aside the prospects of rejecting their omega-knowability.[19] So it better be that at least one of IGNORANCE or Δ-MARGIN isn't omega-knowable. Given OMEGA ASSERTION, this means that at least one of these two principles isn't *assertable*.

This strikes me as something of a surprise, and so evidence against OMEGA ASSERTION. Let's set aside the prospects of denying the assertability of IGNORANCE for now (we'll return to it later). Is there a reasonably principled basis for denying that Δ-MARGIN is assertable?

Well, it doesn't seem at all plausible that Δ-MARGIN might have the status of being an unassertable truth, *a la* a Moorean conjunction like 'p and I don't know that p'. So it better be that it's false: it does *not* follow from the fact that it's determinately the case that a glass that is x% full is pretty full that a glass that is $(x-1)$% full is pretty full.

And perhaps this needn't be such a bitter pill to swallow. We motivated Δ-MARGIN with a rhetorical question: how could it be determinately true that a glass that is 80% full is pretty full, while a glass that is 79% full isn't even pretty full? To the extent this "argument" is persuasive, it's presumably because it channels the following more general thought: if we've got a sorites sequence for F-ness (e.g., *being pretty full*), then any two adjacent members of that sequence are going to be extremely similar in the respects that are relevant to whether they are F. And this in turn makes it hard to see how one member could be determinately F while the next is $\neg F$. Better to think that the determinate Fs are buffered by some non-determinate Fs, before getting to the $\neg F$s (or so the thought goes).

[19] The view that no proposition *whatsoever* is omega-knowable—omega-knowledge *super*-skepticism—is perfectly coherent (see, e.g., Williamson 2002 for relevant discussion). But it's obviously not a live option for someone who endorses OMEGA ASSERTION, like Goldstein, since p_{100} and p_0 are as assertable as anything can be.

The good news for the proponent of OMEGA ASSERTION is that reflection on the logic of omega-knowledge gives us independent reason to doubt that this general thought could be correct. For consider the omega-knowledge analogue of Δ-MARGIN:

ΔK^*-MARGIN: $\Delta K^* p_x \to K^* p_{x-1}$

Assuming you omega-know p_{100}—which I'm happy to assume, and which the proponent of OMEGA ASSERTION *must* assume—ΔK^*-MARGIN cannot be correct. This is because it implies that if you omega-know p_{100}, then you omega-know p_0 (which obviously you don't). Why? Because it follows from IGNORANCE and K^*-POSITIVE that, for any x, if you omega-know p_x, then it's determinate that you omega-know p_x.[20] So if you omega-know p_{100} then it's determinate that you do. But if it's determinate that you omega-know p_{100} then, by ΔK^*-MARGIN, you omega-know p_{99}. But if you omega-know p_{99} then it's determinate that you do, and so by ΔK^*-MARGIN you must omega-know p_{98}... And so on and so forth until we get to the absurd conclusion that (determinately!) you omega-know p_0. So evidently the determinate cases of omega-knowledge needn't be buffered by non-determinate cases of omega-knowledge, for in fact it is impossible for there to be any such cases.

Nonetheless, there are good reasons to believe that the boundaries of our omega-knowledge are vague. This is especially so if one is an omega-knowledge anti-skeptic (like Goldstein), since it is trivial to construct sorites-sequences for the kinds of epistemic states the anti-skeptic takes to constitute omega-knowledge. For example: if I'm in normal conditions looking at what is in fact a 100-foot tall tree, then if I have non-trivial omega-knowledge of anything, I omega-know that the tree is at least 10-feet tall. But suppose the trunk of the tree is put into a massive wood-chipper, which slowly grinds the tree to dust. At some point I'll stop omega-knowing that the tree is at least 10-feet tall. But when exactly does this happen? It can't be the exact moment the tree stops being less than 10-feet tall, since (a) my eyesight is nowhere near good enough to mark that transition and (b), even if it were, there is vagueness in how tall the tree is (*inter alia*). So surely it's vague when I stop omega-knowing that the tree is at least 10-feet tall.

But even the omega-knowledge *skeptic* should think that the boundaries of our omega-knowledge are vague, for this is exactly what is suggested by the case of the emptying glass. You omega-know p_{100}. You don't omega-know p_0. So what is the smallest x such that you omega-know p_x? Once again there seem to be the kind of barriers to answering this question that are distinctive of vagueness. And in this case the barriers have nothing to do with the inexactitude of perception: for any x, it's a purely conceptual matter whether a glass that is x% full is pretty full.[21]

So: omega-knowledge is vague, because sorites-susceptible. Yet ΔK^*-MARGIN is false. So the most straightforward argument for Δ-MARGIN doesn't work, since it can't be a con-

[20] Proof: Suppose $K^* p_x$. Then $KK^* p_x$, by K^*-POSITIVE. But this entails $\Delta K^* p_x$, given IGNORANCE.

[21] One could resist this argument by insisting that, determinately, there is exactly one value of x such that it's omega-knowable that p_x: $x = 100$. But (a) this isn't an option for proponents of OMEGA ASSERTION, since p_{99} is clearly assertable. And (b) it seems independently implausible: if it's (determinately) omega-knowable that a glass that is completely full is pretty full, then surely it's also omega-knowable that a glass that is *just barely less than completely full* is *pretty* full.

(One could also resist the argument by insisting that nothing whatsoever is omega-knowable. But for the reasons given in note 19, this response is dialectically irrelevant.)

ceptual truth about vagueness that determinate Fs are never adjacent to $\neg F$s in sorites-sequences for F-ness. This is good news for the proponent of OMEGA ASSERTION, since we know from above that if OMEGA ASSERTION is true, then at least one of IGNORANCE and Δ-MARGIN must be false. And now there is a path for laying the blame on Δ-MARGIN.

But the bad news for the proponent of OMEGA ASSERTION is that omega-knowledge is vague.[22] For even if this doesn't imply ΔK^*-MARGIN, it most certainly seems to imply ΔK^*-GAP:

ΔK^*-GAP: $\Delta K^* p_x \to \neg\Delta\neg K^* p_{x-1}$

ΔK^*-MARGIN says that a single percentage of fullness can't mark the difference between the determinate presence of omega-knowledge and the absence of omega-knowledge. ΔK^*-GAP, by contrast, says that a single percentage of fullness can't mark the difference between the determinate presence of omega-knowledge and the *determinate* absence of omega-knowledge. ΔK^*-GAP is thus strictly weaker than ΔK^*-MARGIN. And unlike ΔK^*-MARGIN, ΔK^*-GAP strikes me as basically non-negotiable: if ΔK^*-GAP were false, then the question 'At what percentage of fullness do you stop omega-knowing that the glass is pretty full?' would have a determinate answer. But it seems obviously *not* to have a determinate answer, since the matter seems to bear all the ordinary hallmarks of vagueness.[23,24]

[22] Does denying ΔK^*-MARGIN commit one to denying the vagueness of omega-knowledge? Not by itself. (Here my discussion borrows heavily from Bacon 2020.) If we thought omega-knowledge was *negatively* introspective—i.e.:

K^*-NEGATIVE: $\neg K^* p \to K\neg K^* p$

—then we'd have a problem. For we know that $\nabla K^* p_x$ entails $\neg\Delta K^* p_x$, which in turn entails $\neg K^* p_x$. So if K^*-NEGATIVE were true, then $\nabla K^* p_x$ would entail $K\neg K^* p_x$. But given IGNORANCE, that would entail $\Delta\neg K^* p_x$, contradicting $\nabla K^* p_x$. So if nothing else, we have to reject K^*-NEGATIVE. And really this shouldn't be so surprising: if K^*-POSITIVE and K^*-NEGATIVE were both true, then for any x you'd be in a position to know whether you omega-know p_x. And that's simply incompatible with there being an x such that it's vague whether you omega-know p_x—at least given IGNORANCE.

But even with K^*-NEGATIVE set aside, the kind of vagueness exhibited by omega-knowledge (in the context of IGNORANCE) is a bit peculiar. Again, we just established that if it's vague whether you omega-know something, then you don't omega-know it. This doesn't mean that it's not vague what you omega-know. But it does mean that there are no *positive* borderline cases of omega-knowledge—i.e., cases where you in fact have omega-knowledge, just not determinately so. So the vagueness in what you omega-know is always due to the existence of *negative* borderline cases—i.e., cases where you in fact *don't* have omega-knowledge, but not determinately so. This asymmetry is surprising. Relatedly, since there are never any positive borderline cases of omega-knowledge, you can't ever know that it's vague whether you omega-know a particular proposition. After all, given that you know IGNORANCE (and K^*-POSITIVE), you know that if it's vague whether you omega-know p, then you don't omega-know p. So if you know that it's vague whether you omega-know p, then you know that you don't omega-know p. But if you know that you don't omega-know p, then, by IGNORANCE, it's not vague whether you omega-know p. It follows, then, that you can't ever identify any borderline cases of omega-knowledge, since if they were identifiable as such they wouldn't be borderline (cf. Sorensen 1987). This too is surprising.

[23] Though see note 21.

[24] ΔK^*-GAP is so-called because of its resemblance to the "gap" principles discussed in the literature on higher-order vagueness—see, e.g., Wright (1992), Fara (2003), Zardini (2006, 2013a), Fine (2008), Asher *et al.* (2009), Bacon (2020). These principles are meant to capture the plausible idea that there is vagueness at every order. Applied to the case of the emptying glass, the thought is that it's not just vague when the glass stops being pretty full, but vague when it stops being determinate that the glass is pretty full, vague when it stops being determinately determinate that the glass is pretty full, and so on—i.e.:

Δ^i-GAP: $\forall i \in \mathbb{N}(\Delta\Delta^i p_x \to \neg\Delta\neg\Delta^i p_{x-1})$

Given standard assumptions about the logic of determinacy (of the kind we made in §2), it can be shown that

By extension, if it's assertable that it's vague when you stop omega-knowing that the glass is pretty full, then ΔK^*-GAP is assertable. Since the former claim is clearly assertable, so is ΔK^*-GAP. And once again, it's clearly assertable that at the start of the process the glass is pretty full (p_{100}) and that at the end of the process, the glass is not pretty full ($\neg p_0$). So, continuing to assume IGNORANCE is assertable, it follows from OMEGA ASSERTION that p_{100}, $\neg p_0$, IGNORANCE, and ΔK^*-GAP are all omega-knowable.

But now we have once again fallen into inconsistency:[25]

1.	$K^*(\Delta K^* p_x \to \neg\Delta\neg K^* p_{x-1})$	(ΔK^*-GAP)
2.	$\Delta K^* p_{100} \to \neg\Delta\neg K^* p_{99}$	(1)
3.	$K^* p_{100}$	(Assumption)
4.	$\Delta K^* p_{100}$	(K^*-POSITIVE, IGNORANCE, 3)
5.	$\neg\Delta\neg K^* p_{99}$	(2, 4)
6.	$\neg K\neg K^* p_{99}$	(5, IGNORANCE)
7.	$\Diamond K^* p_{99}$	(6)
8.	$\Diamond\Delta K^* p_{99}$	(K^*-POSITIVE, IGNORANCE, 7)
9.	$\Diamond\Delta K^* p_{99} \wedge K^*(\Delta K^* p_{99} \to \neg\Delta\neg K^* p_{98})$	(1, 8)
10.	$\Diamond[\Delta K^* p_{99} \wedge (\Delta K^* p_{99} \to \neg\Delta\neg K^* p_{98})]$	(9)
11.	$\Diamond\neg\Delta\neg K^* p_{98}$	(10)
12.	$\Diamond\neg K\neg K^* p_{98}$	(11, IGNORANCE)
13.	$\Diamond\Diamond K^* p_{98}$	(12)
	$\ldots$	
601.	$\Diamond^{100} K^* p_0$	
602.	$K^*\neg p_0$	(Assumption)
603.	$K^{100} K^*\neg p_0$	(602, K^*-POSITIVE)

Δ^i-GAP cannot be omega-determinate (Fara 2003, Zardini 2006, 2013a). But as Bacon (2020, §8) points out, this is compatible with the claim that Δ^i-GAP could be *giga*-determinate (i.e., n-determinate, for some large n)—which perhaps is enough to account for its appeal.

For the purposes of this paper, we need not take a stand on the status of Δ^i-GAP (true? determinate? giga-determinate?), since it's compatible with my main arguments that there is no such thing as higher-order vagueness (i.e., that for any p and i: $\Delta^i\nabla p \vee \Delta^i\neg\nabla p$). But there is a certain dialectical symmetry between Δ^i-GAP and ΔK^*-GAP that is worth drawing attention to. In particular, we will soon see that if IGNORANCE is omega-knowable, then ΔK^*-GAP cannot be. But it could well be that ΔK^*-GAP is giga-knowable. So one who is warmed up to accepting that Δ^i-GAP is giga-determinate but not omega-determinate may be in a reasonable place to make their peace with the idea that ΔK^*-GAP is giga-knowable despite not being omega-knowable.

[25] This argument is structurally very similar to Zardini's (2013a, §4) "higher-order sorites paradox", which establishes that the omega-determinacy of Δ^i-GAP is incompatible with the omega-determinacy of $p_{100} \wedge \neg p_0$ (see note 24).

604. $K^{100}\neg K^{*}p_0$ (603)

605. $\neg\Diamond^{100}K^{*}p_0$ (604)

606. $\bot$ (601, 605)

Thus, as a matter of (classical) logic, at least one of the following four propositions is not omega-knowable:

- A glass that is completely full is pretty full. (p_{100})
- A glass that is completely empty is not pretty full. ($\neg p_0$)
- Vagueness entails ignorance. (IGNORANCE)
- It's vague when you stop omega-knowing that the glass is pretty full. (ΔK^{*}-GAP)

As before, I'm going to set the aside the prospects of rejecting the omega-knowability of p_{100} or $\neg p_0$. So it's got to be that at least one of IGNORANCE or ΔK^{*}-GAP is not omega-knowable. Which is it?

This is a live question not just for omega-knowledge anti-skeptics like Goldstein, but for anyone who accepts the assumptions about determinacy and knowledge outlined in §2. I myself am not exactly sure of its answer. If forced to say, I'd probably go with: IGNORANCE is omega-knowable while ΔK^{*}-GAP is not.

But thankfully I'm not forced to say. For the purposes of this paper, it actually doesn't matter what the answer to this question is, since the fact that at least one of these two principles is not omega-knowable is enough to establish that OMEGA ASSERTION cannot be true. The simple version of the argument is this: IGNORANCE and ΔK^{*}-GAP are both assertable; IGNORANCE and ΔK^{*}-GAP are not both omega-knowable; so OMEGA ASSERTION is false. Here's the slightly more sophisticated version: IGNORANCE and ΔK^{*}-GAP are both assertable *if true*; IGNORANCE and ΔK^{*}-GAP are both true; IGNORANCE and ΔK^{*}-GAP are not both omega-knowable; so OMEGA ASSERTION is false.

Why think that IGNORANCE and ΔK^{*}-GAP are both assertable if true? Because it would be utterly mysterious why IGNORANCE and ΔK^{*}-GAP should behave like Moorean conjunctions: propositions that might well be true, but that are by their very nature unassertable. After all, the considerations that tell in favor of these principles are entirely open to view. In the case of IGNORANCE, it's the simple observation that when it's vague whether p, it seems impossible to know whether p. In the case of ΔK^{*}-GAP, it's the simple observation that we seem to be able to construct sorites-sequences for omega-knowledge. Perhaps these observations are misleading, or don't actually imply the corresponding principles. But I see no room for either principle to have the status of an unassertable truth.

Thus, supposing IGNORANCE and ΔK^{*}-GAP are both true—as they seem to be!—then we have an assertable proposition of the form 'p, but it's not omega-knowable that p'. I will assert it now: IGNORANCE and ΔK^{*}-GAP are both true, but it's not omega-knowable that IGNORANCE and ΔK^{*}-GAP are both true. By extension, if IGNORANCE and ΔK^{*}-GAP are both true, then OMEGA ASSERTION is false.

In a similar vein, if we make the standard assumption that only that which is knowable is assertable ($Ap \rightarrow Kp$), then this argument also refutes the "KK" principle:

KK: $Kp \rightarrow KKp$

After all, if p_{100}, $\neg p_0$, IGNORANCE, and ΔK^*-GAP are all assertable, then they're all knowable. But given KK, if they're all knowable they're all omega-knowable. They're not all omega-knowable, so KK is false.

Here is where this leaves us. Anyone who accepts OMEGA ASSERTION (or KK) is committed to thinking that at least one of IGNORANCE or ΔK^*-GAP is simply not true (and thus neither assertable nor omega-knowable). For the reasons given earlier, the prospects of rejecting ΔK^*-GAP strike me as quite grim—especially if one is an omega-knowledge anti-skeptic, like Goldstein. So IGNORANCE will have to be what goes. The next section of the paper will explore the prospects of this line.[26]

4 Vagueness without ignorance

Supposing IGNORANCE is false, then we are no longer forced to say that we can't omega-know that it's vague when we stop omega-knowing that the glass is pretty full. This is good news for the proponent of OMEGA ASSERTION, since clearly we can assert that it's vague when we stop omega-knowing that the glass is pretty full.

But as it turns out, whether or not IGNORANCE is true, considerations of vagueness still present a challenge for the proponent of OMEGA ASSERTION. The core reason why is that, as far as the various principles we've considered so far in this paper go, there's very little that separates assertion from knowledge. In particular, all of the main assumptions about the logic of knowledge outlined in §2 are just as plausible as assumptions about the logic of assertion:

$A^*p := p \wedge Ap \wedge AAp \wedge \ldots := \underbrace{A\ldots A}_{\infty}p$. ('It is omega-assertable that p'.)

A-CLOSURE: $A(p \rightarrow q) \rightarrow (Ap \rightarrow Aq)$

A-FACTIVITY: $Ap \rightarrow p$

A-AGGLOMERATION: $(Ap_1 \wedge Ap_2 \wedge \ldots) \rightarrow A(p_1 \wedge p_2 \wedge \ldots)$

[26] I should mention that there is a strictly simpler version of this section's main argument—which I explore in more detail in some work in progress (n.d.)—that does not have IGNORANCE as a premise. This argument establishes that one of the following three propositions is not omega-knowable: (i) p_{100}; (ii) $\neg p_0$; and (iii) that you don't know when you stop omega-knowing that the glass is pretty full, i.e.:

HIDDEN: $KK^*p_x \rightarrow \neg K \neg K^* p_{x-1}$

(Note that HIDDEN is just ΔK^*-GAP with the 'Δ' interpreted as K.)

Though I find HIDDEN independently plausible—especially so conditional on omega-knowledge anti-skepticism—my sense is that the best case for it rests on the conjunction of IGNORANCE and ΔK^*-GAP. Indeed, one who denies IGNORANCE has a fairly principled explanation of why we can't say when we stop omega-knowing when the glass is pretty full, despite the (purported) fact that we *do* know when we stop omega-knowing that the glass is pretty full. And that explanation is that it's vague when we stop omega-knowing when the glass is pretty full, and vagueness is a barrier to assertion (even if not knowledge)—cf. note 28 below.

A^*-POSITIVE: $A^*p \to AA^*p$
A^*-NECESSITATION: $\blacksquare p \to A^*p$

Relatedly, even if we're willing to deny that vagueness is a barrier to knowledge, it seems considerably more difficult to deny that vagueness is a barrier to assertion:

SILENCE: $\neg\Delta p \to \neg Ap$

And this is because if we know anything about vagueness, we know that when it's vague whether p, there's something problematic about going around asserting p without qualification. Someone who sincerely and confidently answers 'Yes' or 'No' to every possible question of the form 'Is a glass that is x% pretty full?' clearly understands at least one of the words in the question other than how we do. So I'll be taking for granted that SILENCE is true in what follows.[27] In fact, I'll be taking for granted that it is assertable, as it obviously seems to be assertable if true.[28]

Now consider the following iteration principle for assertion:

AA: $Ap \to AAp$

AA says that if p is assertable, then it's assertable that p is assertable, assertable that it's assertable that p is assertable, and so on *ad infinitum*. Thus, AA implies that assertability entails omega-assertability ($Ap \to A^*p$). And by the definition of omega-assertion, the reverse direction holds as well: omega-assertability entails assertability ($A^*p \to Ap$). So as far as AA is concerned, there is no distinguishing assertability and omega-assertability ($Ap \leftrightarrow A^*p$).

For reasons we will see in more detail shortly, the proponent of OMEGA ASSERTION faces strong pressure to accept not just that AA is true, but that it is assertable, since: (i) the considerations that tell in favor of OMEGA ASSERTION are essentially the same as those that tell in favor of AA; and (ii) the considerations that tell in favor of OMEGA ASSERTION tell in favor of its being assertable (theorists like Goldstein are happy to go around asserting OMEGA ASSERTION, after all). But I will now argue that AA cannot be true if SILENCE is assertable, for reasons exactly analogous to the reason OMEGA ASSERTION cannot be true if IGNORANCE is assertable.

[27] There are some theorists of vagueness who may be inclined to reject principles like SILENCE. I say 'may' and 'like' because there are delicate hermeneutical issues here, especially in light of my propositional conceptions of vagueness and assertion. But anyway: "contextualists" about vagueness might say that when the sentence p is vague, there will be some contexts in which one may permissibly utter p and some contexts in which one may not permissibly utter p, and that it's (at least sometimes) the speaker's discretion which context they are "in", and thus whether they may permissibly utter p (for discussion, see, e.g., Kamp 1981, Raffman 1994, 1996, Barker 2002, Shapiro 2006, and Carter forthcoming). My sense is that contextualists of this sort ought *not* be hostile to SILENCE, though I lack the space to explore the matter further. "Dialetheists"' about vagueness, on the other hand, are likely to be more straightforwardly hostile to SILENCE. And this is because certain versions of the view lend support to the idea that when the sentence p is vague, speakers may permissibly utter ⌜p∧¬p⌝ (for discussion, see, e.g., Priest 1979, 2010, Hyde 1997, Ripley 2011, 2012, Cobreros *et al.* 2012, Weber *et al.* 2014). I won't be engaging with such views here, since I'd be happy if one of the lessons of the paper is that the proponent of OMEGA ASSERTION ought to be a dialetheist.

[28] Interestingly, Dorr (2003) appeals to SILENCE to explain away some of the evidence for IGNORANCE. According to Dorr, if it's not determinate that p, then it's not determinate that you know that p. Given SILENCE, then, we have a natural explanation of our reluctance to answer affirmatively when asked, of some borderline p, whether we know that p is true.

Assertability is as vague as anything else. Consider again the emptying glass. You can assert p_{100}; you can't assert p_0. So at some point you go from being able to assert that the glass is pretty full to not being able to assert it. But it is vague when this transition occurs. So for no x is it determinate that: you may assert p_x but may not assert p_{x-1}. Thus we have a (first-order)-assertion-theoretic version of the previous section's ΔK^*-GAP:

ΔA-GAP: $\Delta A p_x \to \neg\Delta\neg A p_{x-1}$

Clearly ΔA-GAP is assertable. Thus, given AA, it is omega-assertable. But supposing AA is determinately true—as it must be if it's assertable—then ΔA-GAP entails the corresponding principle about omega-assertion (which is itself must be omega-assertable if ΔA-GAP is):[29]

ΔA^*-GAP: $\Delta A^* p_x \to \neg\Delta\neg A^* p_{x-1}$

Thus: if it's determinate that anything that is assertable is omega-assertable, then if it's vague when you stop being able to assert that the glass is pretty full, it's vague when you stop being able to omega-assert that the glass is pretty full.

But now we can run an argument—exactly analogous to the 606-step one given in the previous section—for the conclusion that at least one of the following four claims is not omega-assertable:[30]

- A glass that is completely full is pretty full. (p_{100})
- A glass that is completely empty is not pretty full. ($\neg p_0$)
- Vagueness entails unassertability. (SILENCE)
- It's vague when you stop being able to omega-assert that the glass is pretty full. (ΔA^*-GAP)

Since all of these claims are omega-assertable if AA is determinately true, it can't be that AA is determinately true. And since it is clear that if AA is true, it's determinately true, it must be that AA is false.[31]

So much the worse for AA. What does any of this have to do with Goldstein's OMEGA ASSERTION? Well, consider the proposition: $p \wedge \neg Ap$. Given A-CLOSURE and A-FACTIVITY, this proposition is not assertable, since $Ap \wedge A\neg Ap$ is inconsistent. But now consider its "dubious" analogue: $p \wedge \neg AAp$. For all we've said, this second proposition *is* assertable, since there is no inconsistency in $Ap \wedge A\neg AAp$. So suppose the proposition that Queen Elizabeth died in 1603 is such that you can assert it, but you can't assert that you can assert

[29] Proof: Suppose $\Delta A^* p_x$. Then $\Delta A p_x$, by A-FACTIVITY. So $\neg\Delta\neg A p_{x-1}$, by ΔA-GAP. But $\Delta(\neg A^* p_{x-1} \to \neg A p_{x-1})$, given that AA is determinate. So by Δ-CLOSURE, we get $\Delta\neg A^* p_{x-1} \to \Delta\neg A p_{x-1}$. Taking the contrapositive, we have: $\neg\Delta\neg A p_{x-1} \to \neg\Delta\neg A^* p_{x-1}$. And thus $\neg\Delta\neg A^* p_{x-1}$.

[30] To run the argument, just uniformly replace the 'K's with 'A's, and interpret '$\Diamond$' as '$\neg A\neg$'. It then proceeds in exactly the same form, with A^*-POSITIVE, SILENCE, and ΔA^*-GAP in place of K^*-POSITIVE, IGNORANCE, and ΔK^*-GAP, respectively.

[31] Note that if AA is false, then if we accept OMEGA ASSERTION, we have to reject the converse of OMEGA ASSERTION—i.e., the thesis that omega-knowledge *suffices* for assertability ($K^*p \to Ap$)—since otherwise we'd have AA. (Proof: Suppose Ap. Then K^*p, by OMEGA ASSERTION. But that's equivalent to K^*K^*p, by K^*-POSITIVE. That gets you K^*Ap, by one application of the converse of OMEGA ASSERTION. A second application gets you AAp.)

it. And suppose that our notion of assertability can be expressed in natural language with ordinary expressions like 'can say that' or 'can tell you that'.[32] Then although you *can't* say:

> ✗ Queen Elizabeth died in 1603, but I can't say that Queen Elizabeth died in 1603.

nothing we have considered so far would stop you from saying:

> ✗ Queen Elizabeth died in 1603, but I can't say whether I can say that Queen Elizabeth died in 1603.

or:

> ✗ Queen Elizabeth died in 1603, but I can't tell you whether I can say whether I can say that Queen Elizabeth died in 1603.

But to my ears, these speeches are exactly as repugnant as the "dubious conjunctions" Goldstein uses to motivate OMEGA ASSERTION, e.g.:

> ✗ Queen Elizabeth died in 1603, but I don't know whether I know that Queen Elizabeth died in 1603.
>
> ✗ Queen Elizabeth died in 1603, but I don't know whether I know whether I know that Queen Elizabeth died in 1603.

Note too that similar points go for discourse:

> [Goldstein's discourse (p. 11)]:
> A: When did Queen Elizabeth die?
> B: She died in 1603.
> A: How do you know you know that?
> B: I didn't say I know I know it.
> A: So you're saying you don't know you know when Queen Elizabeth died?
> B: I'm not saying that either. I'm saying she died in 1603. Maybe I know that I know she died in 1603, maybe I don't. Honestly, I've got no idea. But you didn't ask me about what I know I know, did you? You just asked when she died.
> ...
> [Parody discourse]:
> A: When did Queen Elizabeth die?

[32] Admittedly, the philosopher's notion of *assertability* is semi-technical, and it's not obvious there's any natural language construction that latches onto it perfectly. But English sentences like 'I'm not in a position to tell you that *p*' and 'It wouldn't be fair to say that *p*' seem to express something very close to what we might express with the philosopher's-English sentence 'I am not permitted to assert *p*'. Or at least this is true in the right conversational settings. If I have strong prudential reasons not to say whether *p* (impropriety, illegality, what have you), then I might speak truly in saying 'I can't tell you whether *p*', even if it's abundantly clear that *p* is assertable in the sense of 'is assertable' of interest to philosophers. But if we control for these factors and consider a situation in which my only goal is to be informative, then a speech like 'I can't tell you whether *p*' seems to express exactly what we would express by 'Neither *p* nor $\neg p$ is assertable'. I thus submit that, in such settings, judgments about natural language sentences involving phrases like 'can say that' and 'can tell you that' provide strong evidence about the nature and norms of the semi-technical notion of assertion. And we can easily anchor ourselves into such a setting for the purposes of this paper.

B: She died in 1603.

A: How can you say whether you are able to tell me that?

B: I didn't say I can say whether I am able to tell you that.

A: So you're saying you can't say whether you are able to tell me when Queen Elizabeth died?

B: I'm not saying that either. I'm saying she died in 1603. Maybe I can say that I am able to tell you whether she died in 1603, maybe I can't. Honestly, I've got no idea. But you didn't ask me about what I can say I can tell you, did you? You just asked when she died.

The second discourse sounds no more coherent the first. The lesson Goldstein draws from the first is that assertion requires omega-knowledge. The lesson we'd seemingly have to draw from the second is that assertion requires omega-assertability.[33] But we know from the argument above that we can't draw that lesson, since if we did draw it, then there would be an assertable proposition of the form 'p, but it's not omega-assertable that p'—namely: p_{100}, $\neg p_0$, SILENCE, and ΔA-GAP are all true, but it's not omega-assertable that p_{100}, $\neg p_0$, SILENCE, and ΔA-GAP are all true.[34] And no such proposition is omega-assertable.

So we need some other account of what's going on with the second discourse—an account that will allow us to explain why speeches like 'Queen Elizabeth died in 1603, but I can't say whether I can say that Queen Elizabeth died in 1603' sound bad, despite in principle being assertable.

Finding such an account is no easy task. To give a sense of the issues, note that it won't work to insist merely that $Ap \rightarrow \neg A\neg AAp$, since this principle does nothing to rule out higher-order iterations of the relevant dubious assertions, like $p \wedge \neg AAAp$. To cover *all* such higher-order iterations, we would need a principle along the lines of:[35]

FRAGILE ASSERTION: $Ap \rightarrow \neg A\neg A^{*}p$

But FRAGILE ASSERTION is problematic too, for reasons related to the argument from ΔA^{*}-GAP. Again, that argument establishes that it's a logical truth at least one of p_{100}, $\neg p_0$, SILENCE, and ΔA^{*}-GAP is not omega-assertable. Since logical truths are assertable, it's assertable that at least one of p_{100}, $\neg p_0$, SILENCE, and ΔA^{*}-GAP is not omega-assertable. Thus, by A-AGGLOMERATION and A-CLOSURE, it's assertable that the conjunction of p_{100}, $\neg p_0$, SILENCE, and ΔA^{*}-GAP is not omega-assertable. But given (the contrapositive of) FRAGILE ASSERTION, this entails that the conjunction of p_{100}, $\neg p_0$, SILENCE, and ΔA^{*}-GAP is not assertable, and thus (again by A-AGGLOMERATION and A-CLOSURE) that one of the

[33] I'm suppressing some dialectical subtleties here. It's one thing to allow that $Ap \wedge \neg AAp$; it's another to allow that $A(p \wedge \neg AAp)$. And perhaps proponents of OMEGA ASSERTION can try to find some way of blocking the path from the former to the latter, without invoking the problematic AA. But note that there is an exactly parallel dialectical situation with respect to Goldstein's original argument for OMEGA ASSERTION: it's one thing to allow that $p \wedge \neg KKp$; it's quite another to allow that $A(p \wedge \neg KKp)$. OMEGA ASSERTION is one way of blocking this latter path, but it need not be the only way. In fact, the proponent of OMEGA ASSERTION faces a residual worry here: whatever principle is invoked to block the path from $Ap \wedge \neg AAp$ to $A(p \wedge \neg AAp)$ may well block the path from $p \wedge \neg KKp$ to $A(p \wedge \neg KKp)$, thereby undercutting the abductive case for OMEGA ASSERTION.

[34] Again, if AA is determinately true, then the assertability of ΔA-GAP entails the omega-assertability of ΔA^{*}-GAP.

[35] So-called because of its resemblance to Goldstein's (2024, Ch.3) "Fragility" principle, which says $Kp \rightarrow \neg K\neg K^{*}p$.

conjuncts is not assertable. Since p_{100}, $\neg p_0$, and SILENCE are all clearly assertable, it must be that ΔA^*-GAP is not assertable. I don't see any reasonable path toward the conclusion that ΔA^*-GAP is an unassertable truth, so it must be that ΔA^*-GAP is false. But if ΔA^*-GAP is false, then for some p_x, $\Delta A^* p_x \wedge \Delta\neg A^* p_{x-1}$. But what could this p_x possibly be? And whatever it is, given that we're committed to $\Delta\neg A^* p_{x-1}$, it's hard to see there being any principled grounds for resisting the conclusion that $A\neg A^* p_{x-1}$—which, given FRAGILE ASSERTION, would in turn imply $\neg A p_{x-1}$. So p_x better not be p_{100}, lest we find ourselves on a path to the absurd conclusion that p_{99} is not assertable. But if p_x is something other than p_{100}, then it becomes especially difficult to see how it could be determinate what it is.

In light of these challenges, I submit that dubious assertions are everyone's problem, not just the omega-knowledge skeptic's. Moreover, whatever the final story of their infelicity turns out to be, we have good reason to think that OMEGA ASSERTION will play no role in it. After all, if IGNORANCE is true, then OMEGA ASSERTION is false, and so irrelevant. But if IGNORANCE is false, then omega-knowledge just has very little to do with the question of whether p is assertable, since omega-knowing that p is compatible with p's being vague and thus, by SILENCE, unassertable. It is thus hard to resist the conclusion that OMEGA ASSERTION is either false or an idle wheel.

Does this mean we should be omega-knowledge skeptics? Not necessarily. Omega-knowledge anti-skepticism is in principle compatible with both the truth and falsity of IGNORANCE.[36] But the results of this paper suggest that however we get to omega-knowledge anti-skepticism, it won't be by reflection on what is or is not assertable. Such matters are too vague.

[36] Though again: if IGNORANCE is omega-knowable, then the omega-knowledge anti-skeptic is all but forced to reject KK, and is plain old forced to reject that claim that either Δ-MARGIN or ΔK^*-GAP is omega-knowable.

References

Asher, Nicholas, Dever, Josh, & Pappas, Chris. 2009. Supervaluations Debugged. *Mind*, **118**(472), 901–933.

Bacon, Andrew. 2018. *Vagueness and Thought*. Oxford, England: Oxford University Press.

Bacon, Andrew. 2020. VIII–Vagueness at Every Order. *Proceedings of the Aristotelian Society*, **120**(2), 165–201.

Barker, Chris. 2002. The Dynamics of Vagueness. *Linguistics and Philosophy*, **25**(1), 1–36.

Barnett, David. 2010. Does Vagueness Exclude Knowledge? *Philosophy and Phenomenological Research*, **82**(1), 22–45.

Benton, Matthew A. 2013. Dubious Objections From Iterated Conjunctions. *Philosophical Studies*, **162**(2), 355–358.

Carter, Sam. forthcoming. Keeping Vague Score. *Journal of Philosophy*.

Cobreros, Pablo, Egré, Paul, Ripley, David, & van Rooij, Robert. 2012. Tolerant, Classical, Strict. *Journal of Philosophical Logic*, **41**(2), 347–385.

Cohen, Stewart, & Comesaña, Juan. 2013. Williamson on Gettier Cases and Epistemic Logic. *Inquiry: An Interdisciplinary Journal of Philosophy*, **56**(1), 15–29.

Das, Nilanjan, & Salow, Bernhard. 2018. Transparency and the KK Principle. *Noûs*, **52**(1), 3–23.

Dorr, Cian. 2003. Vagueness Without Ignorance. *Philosophical Perspectives*, **17**(1), 83–113.

Dorr, Cian. 2010. Iterating Definiteness. *In:* Dietz, Richard, & Moruzzi, Sebastiano (eds), *Cuts and clouds: vagueness, its nature, and its logic*. Oxford University Press.

Dorst, Kevin. 2019. Abominable KK Failures. *Mind*, **128**(512), 1227–1259.

Fara, Delia Graff. 2003. Gap Principles, Penumbral Consequence, and Infinitely Higher-Order Vagueness. *In:* Beall, J. C. (ed), *Liars and Heaps: New Essays on Paradox*. Oxford University Press UK.

Fine, Kit. 2008. The Impossibility of Vagueness. *Philosophical Perspectives*, **22**(1), 111–136.

Goldstein, Simon. 2024. *Iterated Knowledge*. Oxford: Oxford University Press.

Gómez-Torrente, Mario. 1997. Two Problems for an Epistemicist View of Vagueness. *Philosophical Issues*, **8**, 237–245.

Gómez-Torrente, Mario. 2002. Vagueness and Margin for Error Principles. *Philosophy and Phenomenological Research*, **64**(1), 107–125.

Greco, Daniel. 2014. Could KK Be OK? *Journal of Philosophy*, **111**(4), 169–197.

Haziza, Eliran. 2022. Assertion and the 'How Do You Know?' Challenge. *Synthese*, **200**(3), 1–17.

Holguín, Ben. On knowing that one doesn't know exactly what one knows. Unpublished manuscript.

Horwich, Paul. 1997. The Nature of Vagueness. *Philosophy and Phenomenological Research*, **57**(4), 929–935.

Hyde, Dominic. 1997. From Heaps and Gaps to Heaps of Gluts. *Mind*, **106**(424), 641–660.

Kamp, Hans. 1981. The Paradox of the Heap. *Journal of Symbolic Logic*, **49**(3), 225–277.

Lee, Christian Ryan. 2016. Excluded Knowledge. *Synthese*, **193**(8), 1–26.

Mandelkern, Matthew, & Dorst, Kevin. 2022. Assertion is Weak. *Philosophers' Imprint*, **22**, 1–20.

Priest, Graham. 1979. The Logic of Paradox. *Journal of Philosophical Logic*, **8**(1), 219–241.

Priest, Graham. 2010. Inclosures, Vagueness, and Self-Reference. *Notre Dame Journal of Formal Logic*, **51**(1), 69–84.

Raffman, Diana. 1994. Vagueness Without Paradox. *Philosophical Review*, **103**(1), 41–74.

Raffman, Diana. 1996. Vagueness and Context-Relativity. *Philosophical Studies*, **81**(2-3), 175–192.

Ripley, David. 2011. Contradictions at the Borders. *Pages 169–188 of:* Nouwen, Rick, van Rooij, Robert, Sauerland, Uli, & Schmitz, Hans-Christian (eds), *Vagueness in Communication*. Springer.

Ripley, David. 2012. Sorting Out the Sorites. *Pages 329–348 of:* Berto, Francesco, Mares, Edwin, Tanaka, Koji, & Paoli, Francesco (eds), *Paraconsistency: Logic and Applications*. Springer.

Shapiro, Stewart. 2006. *Vagueness in Context*. Oxford, England: Oxford University Press UK.

Sorensen, Roy. 1987. The Vagueness of Knowledge. *Canadian Journal of Philosophy*, **17**(4), 767–804.

Sorensen, Roy. 1988. *Blindspots*. New York: Oxford University Press.

Sosa, David. 2009. Dubious Assertions. *Philosophical Studies*, **146**(2), 269–272.

Unger, Peter K. 1975. *Ignorance: A Case for Scepticism*. Oxford University Press.

Weber, Zach, Ripley, David, Priest, Graham, Hyde, Dominic, & Colyvan, Mark. 2014. Tolerating Gluts. *Mind*, **123**(491), 813–828.

Williamson, Timothy. 1994. *Vagueness*. Routledge.

Williamson, Timothy. 2000. *Knowledge and Its Limits*. Oxford University Press.

Williamson, Timothy. 2002. Epistemicist Models: Comments on Gómez-Torrente and Graff. *Philosophy and Phenomenological Research*, **64**(1), 143–150.

Williamson, Timothy. 2013. Response to Cohen, Comesaña, Goodman, Nagel, and Weatherson on Gettier Cases in Epistemic Logic. *Inquiry: An Interdisciplinary Journal of Philosophy*, **56**(1), 77–96.

Wright, Crispin. 1992. Is Higher Order Vagueness Coherent? *Analysis*, **52**(3), 129–139.

Wright, Crispin. 2001. On Being in a Quandary. Relativism Vagueness Logical Revisionism. *Mind*, **110**(1), 45–98.

Yli-Vakkuri, Juhani, & Hawthorne, John. 2022. Being in a Position to Know. *Philosophical Studies*, **179**(4), 1323–1339.

Zardini, Elia. 2006. Squeezing and Stretching: How Vagueness Can Outrun Borderlineness. *Proceedings of the Aristotelian Society*, **106**(3), 419–426.

Zardini, Elia. 2013a. Higher-Order Sorites Paradox. *Journal of Philosophical Logic*, **42**(1), 25–48.

Zardini, Elia. 2013b. Luminosity and Determinacy. *Philosophical Studies*, **165**(3), 765–786.